INTELLIGENT RISK

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Synopsis

This article presents an analysis of the impact of asset price bubbles on the markets for cryptocurrencies and considers the standard risk management measure Value-at-Risk ("VaR"). It applies the theory of local martingales, presents a styled model of asset price bubbles in continuous time and performs a simulation experiment featuring one- and two-dimensional Stochastic Differential Equation ("SDE") systems for asset values through a Constant Elasticity of Variance ("CEV") process that can detect bubble behavior. It summarizes a working research paper that is available from the author upon request, containing mathematical details, complete results and references[1].

measuring risk in new asset classes

by **Michael** Jacobs, Jr, Ph.D., CFA

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The financial crises of the last decades have been the impetus behind a movement to better understand the relative merits of various risk measures, classic examples being Value-at-Risk ("VaR") and related quantities. The importance of an augmented comprehension of these measures is accentuated in the realm of new asset classes such as cryptocurrencies, as observed in the recent meltdown in these markets. We have subsequently learned from episodes such as this that the pricing models have failed in not incorporating the phenomenon of price bubbles, which in turn added to the severity of the downturn for investors and risk managers who mis-measured their potential adverse exposure to market risk in this domain.

The type of asset price bubble considered here exists only in continuous trading models, and it corresponds to an asset whose price process is a local martingale but not a martingale, which is the type of bubble that is the subject of this study. In economic terms, in this case the risk-adjusted expected discounted cash flows and liquidation value at some finite time horizon do not equal the market price, implying that the asset's fundamental value is not equal to its market price. Such bubbles arise when investors attempt to capture short-term trading profits through trading over a finite horizon where the market price for an asset exceeds its fundamental value, the latter being interpreted as the price paid for the asset to buy and hold until liquidation. In this setting, it is possible to test for the existence of such bubbles without estimating an asset's fundamental value, thereby avoiding the joint hypothesis issue.

1 / The working paper may be found on the Social Sciences Research Network: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4399927

Cryptocurrencies are naturally suited to this form of testing since as they have cash flows, and the fundamental value corresponds to the currency's liquidation value at the model's horizon, which implies that bubbles exist in cryptocurrencies when speculators buy to resell before the model's horizon. This situation appears to be rather plausible in the case of novel cryptocurrencies, which are mainly used as a medium of exchange. Theoretically, if purchased to buy and hold and to use as needed, the transaction demand for these assets should be constrained by the usage of other more standard currencies to execute transactions. However, this expectation is at odds with historical experience, as seen in the unprecedented expansion of cryptocurrency markets over the last decade.

testing for asset price bubbles

In this empirical experiment across several widely traded cryptocurrencies, the estimated parameters of one-dimensional SDE systems do not show evidence of bubble behavior. However, when a two-dimensional system is estimated jointly with an equity market index (in this case the NASDAQ), a bubble is detected, and comparing bubble to non-bubble economies, it is shown that asset price bubbles result in materially inflated VaR measures. The implication of this finding for portfolio and risk management is that, rather than acting as a diversifying asset class, cryptocurrencies may not only be highly correlated with other assets but have anti-diversification properties that materially reduce diversification benefits on portfolios.

The estimation results for the one- and two-dimensional SDE models for the case of Bitcoin and the NASDAQ, as well as the simulation of daily VaR for each of these models, are described below. The results and conclusions for the other five cryptocurrencies in the detailed research paper are similar. Table 1 shows that when the SDE is estimated separately for each of them and the NASDAQ, the parameter estimate for the CEV parameter is either statistically indistinguishable from unity, or else is less than one, and the null hypothesis that it exceeds one, which is indicative of no bubble, should be rejected. However, when the two-dimensional systems are estimated, considering the correlation between the cryptocurrency and equity index processes, all CEV parameter estimates are greater than one and enough so that the null hypothesis that it is less than or equal to one, which is evidence of a bubble in the joint price processes, should be rejected. The second major observation is that the normalized VaR measures are materially elevated in the cases of the two-dimensional SDE models where bubble behavior is detected as compared to the one-dimensional case, which holds Bitcoin and the NASDAQ.

			Drift	Vol.	CEV Exp.	Corr.	Log-L	AIC	99th Prcntl. VaR
Bitcoin	1 Dim. SDE	Estimate	0.1012	1.6586	0.9035				
		Std. Err.	0.0249	0.0167	0.0012	N/A	71,643.73	71,644.33	0.8568
	2 Dim. SDE	Estimate	0.9095	1.1167	1.0390	0.9128			
		Std. Err.	0.2784	0.0638	0.0064	0.0282	41,048.81	41,054.81	0.9902
NASDAQ Equity Index	1 Dim. SDE	Estimate	0.0635	0.0565	1.1012				
		Std. Err.	0.0242	0.0022	0.0851	N/A	60,432.10	60,372.10	0.2718
	2 Dim. SDE	Estimate	0.0752	0.0802	1.5196	0.9128			
		Std. Err.	0.0064	0.0000	0.0014	0.0282	41,048.81	41,054.81	0.4930

Table 1: One- and Two-Dimensional SDE System Estimation Results – NASDAQ and Bitcoin

Finally, Figures 1 through 4 below show graphical depictions of the VaR simulations for Bitcoin and the NASDAQ. The extreme non-normality, i.e., extreme excess kurtosis and skewness, of the simulated loss distribution in the case of the two- versus the one-dimensional model are evident from these plots.

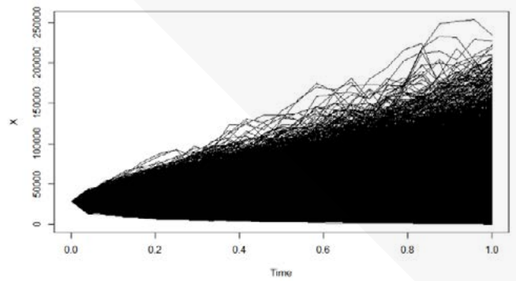

Figure 1: Simulation of One-Day VaR from the Estimation of a One-Dimensional SDE System CEV Model – Bitcoin

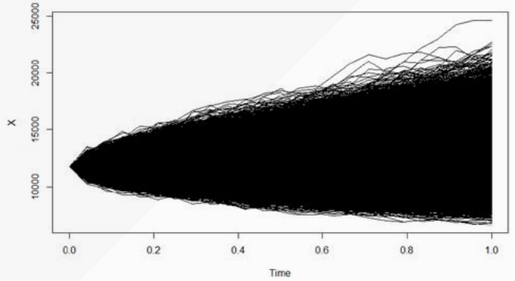

Figure 2: Simulation of One-Day VaR from the Estimation of a One-Dimensional SDE System CEV Model – NASDAQ

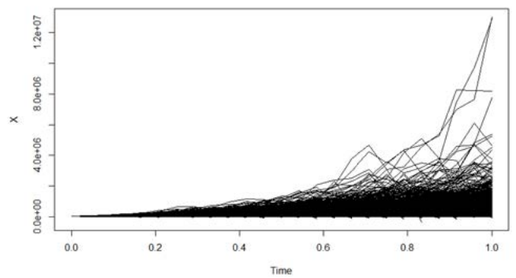

Figure 3: Simulation of One-Day VaR from the Estimation of a Two-Dimensional SDE System CEV Model – Bitcoin

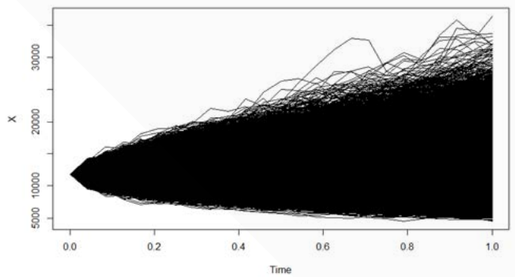

Figure 4: Simulation of One-Day VaR from the Estimation of a Two-Dimensional SDE System CEV Model – NASDAQ

conclusion

Leveraging the deep economic literature of local martingale theory as applied to asset price bubbles in the markets for cryptocurrencies and using historical time series data in a continuous time and finite horizon trading model setting has produced an important finding. The example with Bitcoin and the NASDAQ showed that asset price bubbles are detected in CEV model dynamics derived from calibration of two- versus one- dimensional SDE models where cryptocurrencies are modeled jointly with an equity price index. We note implications of this research for prudential supervision and public policy and the debate over how cryptocurrencies should be regulated, namely that, if there is a powerful interaction between cryptocurrencies and another major risk asset that leads to a self-reinforcing vicious cycle of bubble behavior, the regulatory regime should account for these linkages and that there should be a proper coordination amongst agencies.

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